

ĐỀ 1

Câu 1 (4,0đ): Cho $\sin \alpha = \frac{4}{5}$ và $0 < \alpha < \frac{\pi}{2}$

a. Tính các giá trị $\cos \alpha, \tan \alpha, \cot \alpha$.

b. Tính $A = \sin(\alpha + \pi) + \cos(-\alpha) + \tan\left(\alpha + \frac{\pi}{2}\right) + \cot\left(\frac{\pi}{2} - \alpha\right)$.

Câu 2 (2,0đ): Chứng minh rằng: $\cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right) = -\cos x$.

Câu 3 (3,0đ):

a. Rút gọn biểu thức $B = \frac{\cos x + \cos 3x + \cos 5x + \cos 7x}{\sin x + \sin 3x + \sin 5x + \sin 7x}$.

b. Chứng minh biểu thức sau không phụ thuộc vào x:

$$C = \sin^2 x + \sin^2\left(\frac{5\pi}{3} + x\right) + \sin^2\left(\frac{5\pi}{3} - x\right) - 4 \tan\left(x - \frac{\pi}{3}\right) \cdot \tan\left(x + \frac{7\pi}{6}\right).$$

Câu 4 (1,0đ): Cho tam giác ABC thỏa mãn $\frac{\sin^2 A}{\cos A} + \frac{\sin^2 B}{\cos B} = (\sin A + \sin B) \cot \frac{C}{2}$.

Chứng minh tam giác ABC cân.

ĐỀ 2

Câu 1 (4,0đ): Cho $\cos \alpha = \frac{4}{5}$ và $0 < \alpha < \frac{\pi}{2}$

a. Tính các giá trị $\sin \alpha, \tan \alpha, \cot \alpha$.

b. Tính $A = \sin(\pi - \alpha) + \cos(\pi + \alpha) + \tan(-\alpha) + \cot\left(\alpha + \frac{\pi}{2}\right)$.

Câu 2 (2,0đ): Chứng minh rằng: $\sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right) = -\sin x$.

Câu 3 (3,0đ):

a. Rút gọn biểu thức $B = \frac{\sin 2x + \sin 4x + \sin 6x + \sin 8x}{\cos 2x + \cos 4x + \cos 6x + \cos 8x}$.

b. Chứng minh biểu thức sau không phụ thuộc vào x:

$$C = \cos^2 x + \cos^2\left(\frac{4\pi}{3} + x\right) + \cos^2\left(\frac{4\pi}{3} - x\right) - 4 \tan\left(x - \frac{\pi}{6}\right) \cdot \tan\left(x + \frac{7\pi}{3}\right).$$

Câu 4 (1,0đ): Cho tam giác ABC thỏa mãn $\frac{\sin^2 A}{\cos A} + \frac{\sin^2 C}{\cos C} = (\sin A + \sin C) \cot \frac{B}{2}$.

Chứng minh tam giác ABC cân.

ĐÁP ÁN HƯỚNG DẪN CHẤM

ĐỀ 1

CÂU	NỘI DUNG	ĐIỂM
Câu 1a (2.0đ)	Ta có : $\cos^2 \alpha = 1 - \sin^2 \alpha = \frac{9}{25} \Rightarrow \cos \alpha = \pm \frac{3}{5}$	0.5
	Vì $0 < \alpha < \frac{\pi}{2}$ nên $\cos \alpha = \frac{3}{5}$	0.5
	Do đó $\tan \alpha = \frac{4}{3}, \cot \alpha = \frac{3}{4}$	0.5+0.5
Câu 1b (2.0đ)	Tính	
	$A = \sin(\alpha + \pi) + \cos(-\alpha) + \tan\left(\alpha + \frac{\pi}{2}\right) + \cot\left(\frac{\pi}{2} - \alpha\right) =$ $= -\sin \alpha + \cos \alpha - \cot \alpha + \tan \alpha = \frac{-4}{5} + \frac{3}{5} - \frac{3}{4} + \frac{4}{3} = \frac{23}{60}$	0.5+0.5 0.5+0.5
Câu 2 (2.0đ)	$VT = \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right) = 2 \cos x \cos \frac{2\pi}{3} = -\cos x = VP$	1+1
Câu 3a (2.0đ)	$a) B = \frac{\cos x + \cos 3x + \cos 5x + \cos 7x}{\sin x + \sin 3x + \sin 5x + \sin 7x} =$ $= \frac{(\cos x + \cos 7x) + (\cos 3x + \cos 5x)}{(\sin x + \sin 7x) + (\sin 3x + \sin 5x)} = \frac{2 \cos 4x \cos 3x + 2 \cos 4x \cos x}{2 \sin 4x \cos 3x + 2 \sin 4x \cos x} =$ $= \frac{\cos 4x}{\sin 5x} = \cot 4x$	1 1
Câu 3b (1.0đ)	$b) \text{ Ta có : } \tan\left(x + \frac{7\pi}{6}\right) = \tan\left(x - \frac{\pi}{3} + \frac{3\pi}{2}\right) = -\cot\left(x - \frac{\pi}{3}\right)$	
	$\text{Do đó : } \tan\left(x - \frac{\pi}{3}\right) \cdot \tan\left(x + \frac{7\pi}{6}\right) = -1$ $\sin^2 x + \sin^2\left(\frac{5\pi}{3} + x\right) + \sin^2\left(\frac{5\pi}{3} - x\right) =$ $= \frac{1 - \cos 2x}{2} + \frac{1 - \cos(10\pi/3 + 2x)}{2} + \frac{1 - \cos(10\pi/3 - 2x)}{2} =$ $= \frac{3}{2} - \frac{\cos 2x + 2 \cos(10\pi/3) \cos 2x}{2} = \frac{3}{2}$ $\text{Suy ra } C = \frac{3}{2} + 4 = \frac{9}{2}$	0.5 0.5

Câu 4 (1.0đ)	Ta có : $\frac{\sin^2 A}{\cos A} + \frac{\sin^2 B}{\cos B} = (\sin A + \sin B) \cot \frac{C}{2}$ $\Leftrightarrow \sin A \tan A + \sin B \tan B = \sin A \cdot \tan\left(\frac{A+B}{2}\right) + \sin B \cdot \tan\left(\frac{A+B}{2}\right)$ $\Leftrightarrow \sin A \left(\tan A - \tan\left(\frac{A+B}{2}\right) \right) + \sin B \left(\tan B - \tan\left(\frac{A+B}{2}\right) \right) = 0$	0.25
	$\Leftrightarrow \sin A \frac{\sin\left(\frac{A-B}{2}\right)}{\cos A \cos\left(\frac{A+B}{2}\right)} + \sin B \frac{\sin\left(\frac{B-A}{2}\right)}{\cos B \cos\left(\frac{A+B}{2}\right)} = 0$	0.25
	$\Leftrightarrow \tan A \sin\left(\frac{A-B}{2}\right) - \tan B \sin\left(\frac{A-B}{2}\right) = 0$	0.25
	$\Leftrightarrow \begin{cases} \tan A = \tan B \\ \sin\left(\frac{A-B}{2}\right) \end{cases} \Leftrightarrow A = B$	0.25
	Vậy tam giác ABC cân tại C.	

ĐÁP ÁN HƯỚNG DẪN CHẤM

Đề 2

CÂU	NỘI DUNG	ĐIỂM
Câu 1a (2.0đ)	Ta có : $\sin^2 \alpha = 1 - \cos^2 \alpha = \frac{9}{25} \Rightarrow \sin \alpha = \pm \frac{3}{5}$	0.5
	Vì $0 < \alpha < \frac{\pi}{2}$ nên $\sin \alpha = \frac{3}{5}$	0.5
	Do đó $\tan \alpha = \frac{3}{4}, \cot \alpha = \frac{4}{3}$	0.5+0.5
Câu 1b (2.0đ)	Tính	
	$A = \sin(\pi - \alpha) + \cos(\pi + \alpha) + \tan(-\alpha) + \cot\left(\alpha + \frac{\pi}{2}\right) =$ $= \sin \alpha - \cos \alpha - \tan \alpha - \tan \alpha = \frac{3}{5} - \frac{4}{5} - 2 \cdot \frac{3}{4} = \frac{-17}{10}$	0.5+0.5 0.5+0.5
Câu 2 (2.0đ)	$VT = \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right) = 2 \sin x \cos \frac{2\pi}{3} = -\sin x = VP$	1+1
Câu 3a (2.0đ)	$a) B = \frac{\sin 2x + \sin 4x + \sin 6x + \sin 8x}{\cos 2x + \cos 4x + \cos 6x + \cos 8x} =$ $= \frac{(\sin 2x + \sin 8x) + (\sin 6x + \sin 4x)}{(\cos 2x + \cos 8x) + (\cos 6x + \cos 4x)} = \frac{2 \sin 5x \cos 3x + 2 \sin 5x \cos x}{2 \cos 5x \cos 3x + 2 \cos 5x \cos x} =$ $= \frac{\sin 5x}{\cos 5x} = \tan 5x$	1 1
Câu 3b (1.0đ)	$b) \text{ Ta có : } \tan\left(x + \frac{7\pi}{3}\right) = \tan\left(x - \frac{\pi}{6} + \frac{5\pi}{2}\right) = -\cot\left(x - \frac{\pi}{6}\right)$ $\text{Do đó : } \tan\left(x - \frac{\pi}{6}\right) \cdot \tan\left(x + \frac{7\pi}{3}\right) = -1$ $\cos^2 x + \cos^2\left(\frac{5\pi}{3} + x\right) + \cos^2\left(\frac{5\pi}{3} - x\right) =$ $= \frac{1 + \cos 2x}{2} + \frac{1 + \cos(10\pi/3 + 2x)}{2} + \frac{1 + \cos(10\pi/3 - 2x)}{2} =$ $= \frac{3}{2} + \frac{\cos 2x + 2 \cos(10\pi/3) \cos 2x}{2} = \frac{3}{2}$ $\text{Suy ra } C = \frac{3}{2} + 4 = \frac{9}{2}$	0.5 0.5

Câu 4 (1.0đ)	Ta có : $\frac{\sin^2 A}{\cos A} + \frac{\sin^2 C}{\cos C} = (\sin A + \sin C) \cot \frac{B}{2}$ $\Leftrightarrow \sin A \tan A + \sin C \tan C = \sin A \cdot \tan\left(\frac{A+C}{2}\right) + \sin C \cdot \tan\left(\frac{A+C}{2}\right)$ $\Leftrightarrow \sin A \left(\tan A - \tan\left(\frac{A+C}{2}\right) \right) + \sin C \left(\tan C - \tan\left(\frac{A+C}{2}\right) \right) = 0$ $\Leftrightarrow \sin C \frac{\sin\left(\frac{A-C}{2}\right)}{\cos A \cos\left(\frac{A+C}{2}\right)} + \sin C \frac{\sin\left(\frac{C-A}{2}\right)}{\cos B \cos\left(\frac{A+C}{2}\right)} = 0$ $\Leftrightarrow \tan A \sin\left(\frac{A-C}{2}\right) - \tan C \sin\left(\frac{A-C}{2}\right) = 0$ $\Leftrightarrow \begin{cases} \tan A = \tan C \\ \sin\left(\frac{A-C}{2}\right) = 0 \end{cases} \Leftrightarrow A = C$ Vậy tam giác ABC cân tại B.	0.25
		0.25
		0.25
		0.25
		0.25

